

Ex 1

$$\begin{aligned}
 \underline{2.} \quad \vec{V}_{O_2/O} &= \left. \frac{d \vec{OO}_2}{dt} \right|_0 = \left. \frac{d (\vec{OO}_1 + \vec{O}_1 O_2)}{dt} \right|_0 \\
 &= \left. \frac{d (\lambda(t) \vec{x}_1 + a_1 \vec{x}_1 + b_1 \vec{y}_1)}{dt} \right|_0 \\
 &= \left. \frac{d ((\lambda(t) + a_1) \vec{x}_1 + b_1 \vec{y}_1)}{dt} \right|_0 \\
 &= \lambda \vec{x}_1 + (\lambda(t) + a_1) \left. \frac{d \vec{x}_1}{dt} \right|_0 + b_1 \left. \frac{d \vec{y}_1}{dt} \right|_0
 \end{aligned}$$

$$\left. \frac{d \vec{x}_1}{dt} \right|_0 = \left. \frac{d \vec{x}_1}{dt} \right|_1 + \vec{\Omega}_{1/O} \wedge \vec{x}_1$$

$$= \vec{0} + \dot{\alpha} \vec{z} \wedge \vec{x}_1 = \dot{\alpha} \vec{y}_1 = 0 \quad \text{car } \alpha = \text{cte}$$

$$\vec{V}_{O_2/O} = \lambda(t) \vec{x}_1$$

$$\begin{aligned}
 \vec{V}_{G/O} &= \left. \frac{d \vec{OG}}{dt} \right|_0 = \left. \frac{d \vec{OO}_2}{dt} \right|_0 + \left. \frac{d \vec{O}_2 G}{dt} \right|_0 \\
 &= \lambda(t) \vec{x}_1 + \left. \frac{d (a_2 \vec{x}_2)}{dt} \right|_0 \\
 &= \lambda(t) \vec{x}_1 + a_2 \left. \frac{d \vec{x}_2}{dt} \right|_0
 \end{aligned}$$

$$\left. \frac{d \vec{x}_2}{dt} \right|_0 = \left. \frac{d \vec{x}_2}{dt} \right|_2 + \vec{\Omega}_{2/O} \wedge \vec{x}_2$$

$$= (\dot{\alpha} + \dot{\beta}) \vec{z} \wedge \vec{x}_2 = \dot{\beta} \vec{y}_2$$

$$\vec{V}_{G/O} = \lambda(t) \vec{x}_1 + \dot{\beta} a_2 \vec{y}_2$$

$$\begin{aligned}
 \underline{3.} \quad \vec{a}_{G/O} &= \left. \frac{d \vec{V}_{G/O}}{dt} \right|_0 = \ddot{\lambda} \vec{x}_1 + \ddot{\beta} a_2 \vec{y}_2 + \dot{\beta} a_2 \left. \frac{d \vec{y}_2}{dt} \right|_0 \\
 &= \ddot{\lambda} \vec{x}_1 + \ddot{\beta} a_2 \vec{y}_2 + \dot{\beta} a_2 (\vec{\Omega}_{2/O} \wedge \vec{y}_2) \\
 &= \ddot{\lambda} \vec{x}_1 + \ddot{\beta} a_2 \vec{y}_2 + \dot{\beta} a_2 (-\vec{x}_2)
 \end{aligned}$$

$$\underline{4.} \quad \vec{x}_1 \cdot \vec{y}_2 = -\sin \beta \quad \text{donc}$$

Ex 2

$$\underline{1.} \quad \overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} \\ = h \vec{z}_0^\circ + \lambda(t) \vec{x}_0^\circ + R \vec{x}_2^\circ$$

$$\underline{2.} \quad \overrightarrow{\Omega_{1/0}} = \vec{0}$$

$$\overrightarrow{\Omega_{2/1}} = \dot{\Theta} \vec{z}^\circ$$

$$\overrightarrow{\Omega_{2/0}} = \ddot{\Theta} \vec{z}^\circ$$

$$\underline{3.} \quad \overrightarrow{V_{H/0}} = \frac{d \overrightarrow{OM}}{dt} \Big|_0 = \frac{d(h \vec{z}_0^\circ + \lambda(t) \vec{x}_0^\circ + R \vec{x}_2^\circ)}{dt} \Big|_0$$

$$= \dot{\lambda}(t) \vec{x}_0^\circ + R \frac{d \vec{x}_2^\circ}{dt} \Big|_0$$

$$= \dot{\lambda} \vec{x}_0^\circ + R \cdot \overrightarrow{\Omega_{2/0}} \wedge \vec{x}_2^\circ$$

$$= \dot{\lambda} \vec{x}_0^\circ + R \cdot \dot{\Theta} \vec{g}_2^\circ \wedge \vec{x}_2^\circ$$

$$= \dot{\lambda} \vec{x}_0^\circ + R \dot{\Theta} \vec{g}_2^\circ = \dot{\lambda} \vec{x}_0^\circ + R \omega \vec{g}_2^\circ$$

$$\underline{5.} \quad \overrightarrow{V_{H/0}} = \dot{\lambda} \vec{x}_0^\circ + R \omega (-\sin \Theta \vec{g}_0^\circ + \cos \Theta \vec{g}_2^\circ)$$

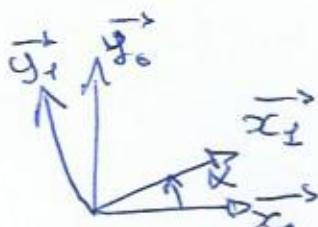
Max pour $\Theta = \frac{\pi}{2}$. On a alors

$$V_{\text{Max}} = \dot{\lambda} + R \omega.$$

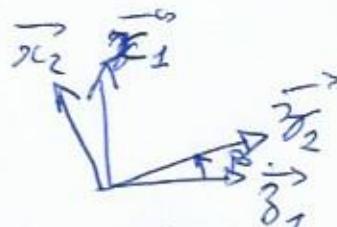
$$\underline{6.} \quad \dot{\lambda} = V_{\text{Max}} - R \omega$$

Ex 3

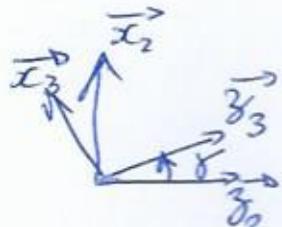
1.



$$\overrightarrow{\Omega_{1/0}} = \dot{\lambda} \vec{z}_0^\circ$$



$$\overrightarrow{\Omega_{2/1}} = \dot{\beta} \vec{g}_1^\circ$$



2.

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$$

$$= h \vec{z}_0^\circ + H \vec{z}_2^\circ + L \vec{z}_3^\circ$$

$$= h \vec{z}_0^\circ + H (\cos \beta \vec{g}_1^\circ + \sin \beta \vec{g}_2^\circ) +$$

$$L (\cos \gamma \vec{g}_2^\circ + \sin \gamma \vec{g}_3^\circ)$$

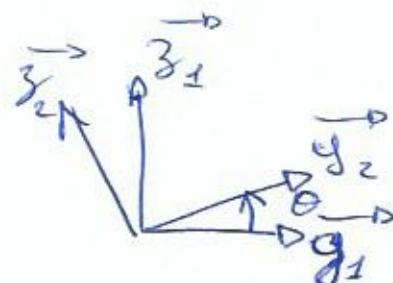
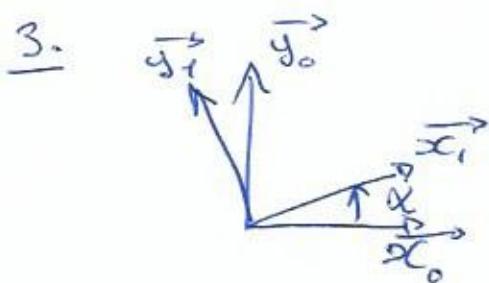
$$\begin{aligned}
 \vec{OC} &= h \vec{z}_0 + H(\cos \beta \vec{z}_0 + \sin \beta \cos \alpha \vec{z}_0^\circ + \sin \beta \sin \alpha \vec{x}_0^\circ) \quad (3) \\
 &\quad + L(\cos \gamma \cos \beta \vec{z}_0 + \cos \gamma \sin \beta \cos \alpha \vec{z}_0^\circ + \\
 &\quad \cos \gamma \sin \beta \sin \alpha \vec{x}_0^\circ + \sin \gamma \cos \beta \vec{x}_1^\circ + \\
 &\quad \sin \gamma \sin \beta \vec{z}_1^\circ) \\
 &= h \vec{z}_0 + H(\cos \beta \vec{z}_0 + \sin \beta \cos \alpha \vec{z}_0^\circ + \sin \beta \sin \alpha \vec{x}_0^\circ) + \\
 &\quad L(\cos \gamma \cos \beta \vec{z}_0 + \cos \gamma \sin \beta \cos \alpha \vec{z}_0^\circ + \\
 &\quad \cos \gamma \sin \beta \sin \alpha \vec{x}_0^\circ + \sin \gamma \cos \beta (\cos \alpha \vec{x}_0 + \sin \alpha \vec{y}_0) - \\
 &\quad \sin \gamma \sin \beta \vec{z}_1^\circ) \\
 &= \begin{cases} H \sin \beta \sin \alpha + L \cos \gamma \sin \beta \sin \alpha + L \sin \gamma \cos \beta \cos \alpha \\ L \sin \gamma \cos \beta \sin \alpha \\ 0 \end{cases} \\
 &\quad h + H \cos \beta + H \sin \beta \cos \alpha + L \cos \gamma \cos \beta + L \cos \gamma \sin \beta \cos \alpha
 \end{aligned}$$

$$\underline{3.} \quad \vec{V}_{c_0} = \begin{cases} H \dot{\beta} \cos \beta \sin \alpha + H \dot{\alpha} \sin \beta \cos \alpha + L (-\dot{\gamma} \sin \beta \sin \alpha \sin \theta \cos \beta \\ + L \dot{\gamma} \sin \gamma \cos \beta \sin \alpha \sin \theta \cos \beta + \dot{\beta} \sin \gamma \sin \beta \cos \alpha + \dot{\gamma} \cos \gamma \cos \beta \cos \alpha) \\ 4 \dot{\gamma} \cos \gamma \cos \beta \sin \alpha + \dot{\alpha} \sin \beta \cos \beta \cos \alpha - \dot{\beta} \sin \beta \sin \beta \sin \alpha \end{cases}$$

Ex 4

1. Rotations

2. Circles



4. $\vec{\Omega}_{10} = \dot{\alpha} \vec{z}_0$

$$\vec{\Omega}_{21} = \dot{\theta} \vec{x}_1$$

5. $\vec{\Omega}_{20} = \vec{\Omega}_{21} + \vec{\Omega}_{10}$

$$= \dot{\alpha} \vec{z}_0 + \dot{\theta} \vec{x}_1$$

(4)

$$\begin{aligned}
 6. \quad \vec{V}_{G_2/10} &= \frac{d \vec{OG_2}}{dt} \Big|_0 \\
 &= \frac{d(\vec{OB} + \vec{BG_2})}{dt} \Big|_0 \\
 &= \frac{d(b\vec{j}_2 + a\vec{x}_1)}{dt} \Big|_0 \\
 &= b \frac{d\vec{j}_2}{dt} \Big|_0 + a \frac{d\vec{x}_1}{dt} \Big|_0 \\
 &= b \cdot \vec{\Omega}_{2/10} \wedge \vec{j}_2 + a \vec{\Omega}_{2/10} \wedge \vec{x}_1 \\
 &= b \cdot (\dot{\alpha} \vec{j}_0 + \dot{\theta} \vec{x}_1) \wedge \vec{j}_2 + a \dot{\theta} \vec{x}_1 \wedge \vec{x}_1 \\
 &= -b \cdot \ddot{\alpha} \sin \theta \vec{j}_2 + b \ddot{\theta} (-\vec{j}_2) \\
 \vec{j}_0 &= \cos \theta \vec{j}_2 - \sin \theta \vec{y}_2
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \frac{d \vec{V}_{G_2/10}}{dt} \Big|_0 &= \vec{a}_{G_2/10} \\
 &= -b \ddot{\alpha} \sin \theta \vec{x}_2 - b \ddot{\alpha} \dot{\theta} \sin \theta \vec{x}_2 - b \ddot{\theta} \vec{y}_2 \\
 &\quad + b \ddot{\alpha} \sin \theta \frac{d \vec{x}_2}{dt} \Big|_0 + b \ddot{\theta} (-\vec{j}_2) - b \ddot{\theta} \frac{d \vec{j}_2}{dt} \\
 \frac{d \vec{x}_2}{dt} \Big|_0 &= \vec{\Omega}_{2/10} \wedge \vec{x}_2 = (\dot{\alpha} \vec{j}_0 + \dot{\theta} \vec{x}_1) \wedge \vec{x}_2 \\
 &= \dot{\alpha} \vec{j}_0 \wedge \vec{x}_2 = \dot{\alpha} \vec{y}_1 \\
 \frac{d \vec{j}_2}{dt} \Big|_0 &= \vec{\Omega}_{2/10} \wedge \vec{j}_2 = (\dot{\alpha} \vec{j}_0 + \dot{\theta} \vec{x}_1) \wedge \vec{j}_2 \\
 &= -\dot{\alpha} \cos \theta \vec{x}_2 + \dot{\theta} \vec{j}_2 \\
 \vec{a}_{G_2/10} &= -b \ddot{\alpha} \sin \theta \vec{x}_2 - b \ddot{\alpha} \dot{\theta} \sin \theta \vec{x}_2 - \\
 &\quad b \ddot{\alpha} \sin \theta (\dot{\alpha} \vec{y}_1) + b \ddot{\theta} (-\vec{j}_2) - b \ddot{\theta} (-\alpha \cos \theta \vec{x}_2 + \dot{\theta} \vec{j}_2)
 \end{aligned}$$