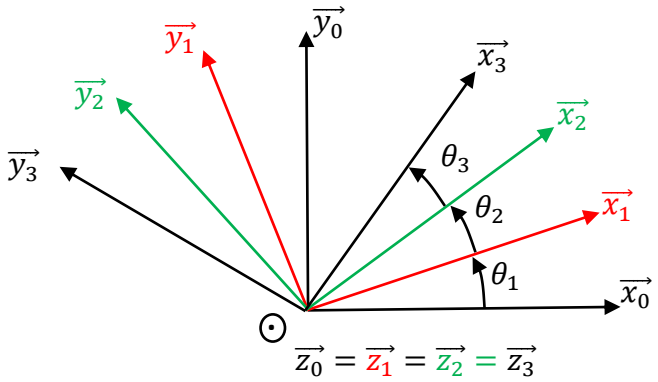
Exercice 2 : Robot ramasseur de fruits

Question 1 :



Question 2: $\overrightarrow{0_0M} = \overrightarrow{0_00_1} + \overrightarrow{0_10_2} + \overrightarrow{0_2M} = l_1\,\overrightarrow{x_1} + l_2\,\overrightarrow{x_2} + l_3\,\overrightarrow{x_3}$

Question 3: $\overrightarrow{0_0M}.\overrightarrow{x_0} = 1 <=> (l_1\,\overrightarrow{x_1} + l_2\,\overrightarrow{x_2} + l_3\,\overrightarrow{x_3})\,.\overrightarrow{x_0} = 1$

$$<=> l_1\cos(\theta_1) + l_2\,\cos(\theta_1+\theta_2) + l_3\,\cos(\theta_1+\theta_2+\theta_3) = 1$$

Question 4:

$$\cos(\theta_1+\theta_2+\theta_3) = \frac{1 - l_1\cos(\theta_1) - l_2\,\cos(\theta_1+\theta_2)}{l_3}$$

$$<=> \theta_3 = \cos^{-1}\left(\frac{1 - l_1\cos(\theta_1) - l_2\,\cos(\theta_1+\theta_2)}{l_3}\right) - \theta_1 - \theta_2$$

$$<=> \theta_3 = \cos^{-1}\left(\frac{1 - 2\cos(30°) - 2\,\cos(150°)}{2}\right) - 150 = -148°$$

Question 5: $\left\|\overrightarrow{0_0M}\right\| = cste$

$$<=> \sqrt{\overrightarrow{0_0M}.\overrightarrow{0_0M}} = cste$$

$$<=> \sqrt{(l_1\,\overrightarrow{x_1} + l_2\,\overrightarrow{x_2} + l_3\,\overrightarrow{x_3})\,.(l_1\,\overrightarrow{x_1} + l_2\,\overrightarrow{x_2} + l_3\,\overrightarrow{x_3})} = cste$$

$$<=> \sqrt{{l_1}^2 + {l_2}^2 + {l_3}^2 + 2(l_1l_2\cos(\theta_2) + l_2l_3\cos(\theta_3) + l_1l_3\cos(\theta_2+\theta_3))} = cste$$

Question 6: $\vec{\Omega}_{1/0} = \dot{\theta}_1\,\overrightarrow{z_1}$ $\vec{\Omega}_{2/1} = \dot{\theta}_2\,\overrightarrow{z_2}$ $\vec{\Omega}_{3/2} = \dot{\theta}_3\,\overrightarrow{z_3}$

$$\vec{\Omega}_{3/0} = \vec{\Omega}_{3/2} + \vec{\Omega}_{2/1} + \vec{\Omega}_{1/0} = (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\,\overrightarrow{z_1}$$

Question 7 : $\vec{V}_{0_0\in 1/0} = \vec{V}_{0_1\in 2/1} = \vec{V}_{0_2\in 3/2} = \vec{0}$

Question 8 :

$$\vec{V}_{M\in 1/0} = \vec{V}_{0_0\in 1/0} + \overrightarrow{M0_0} \wedge \vec{\Omega}_{1/0} = \vec{0} - (l_1\,\overrightarrow{x_1} + l_2\,\overrightarrow{x_2} + l_3\,\overrightarrow{x_3}) \wedge \dot{\theta}_1\,\overrightarrow{z_1} = (l_1\,\overrightarrow{y_1} + l_2\,\overrightarrow{y_2} + l_3\,\overrightarrow{y_3})\dot{\theta}_1$$

$$\vec{V}_{M\in 2/1} = \vec{V}_{0_1\in 2/1} + \overrightarrow{M0_1} \wedge \vec{\Omega}_{2/1} = \vec{0} - (l_2\,\overrightarrow{x_2} + l_3\,\overrightarrow{x_3}) \wedge \dot{\theta}_2\,\overrightarrow{z_1} = (l_2\,\overrightarrow{y_2} + l_3\,\overrightarrow{y_3})\dot{\theta}_2$$

$$\vec{V}_{M\in 3/2} = \vec{V}_{0_2\in 3/2} + \overrightarrow{M0_2} \wedge \vec{\Omega}_{3/2} = \vec{0} - l_3\,\overrightarrow{x_3} \wedge \dot{\theta}_3\,\overrightarrow{z_1} = \dot{\theta}_3\,l_3\,\overrightarrow{y_3}$$

Question 9 : $\{V_{1/0}\} = \begin{Bmatrix} \dot{\theta}_1\,\overrightarrow{z_1} \\ (l_1\,\overrightarrow{y_1} + l_2\,\overrightarrow{y_2} + l_3\,\overrightarrow{y_3})\dot{\theta}_1 \end{Bmatrix}_M$

$\{V_{2/1}\} = \begin{Bmatrix} \dot{\theta}_2\,\overrightarrow{z_2} \\ (l_2\,\overrightarrow{y_2} + l_3\,\overrightarrow{y_3})\dot{\theta}_2 \end{Bmatrix}_M$ $\qquad$ $\{V_{3/2}\} = \begin{Bmatrix} \dot{\theta}_3\,\overrightarrow{z_3} \\ \dot{\theta}_3\,l_3\,\overrightarrow{y_3} \end{Bmatrix}_M$

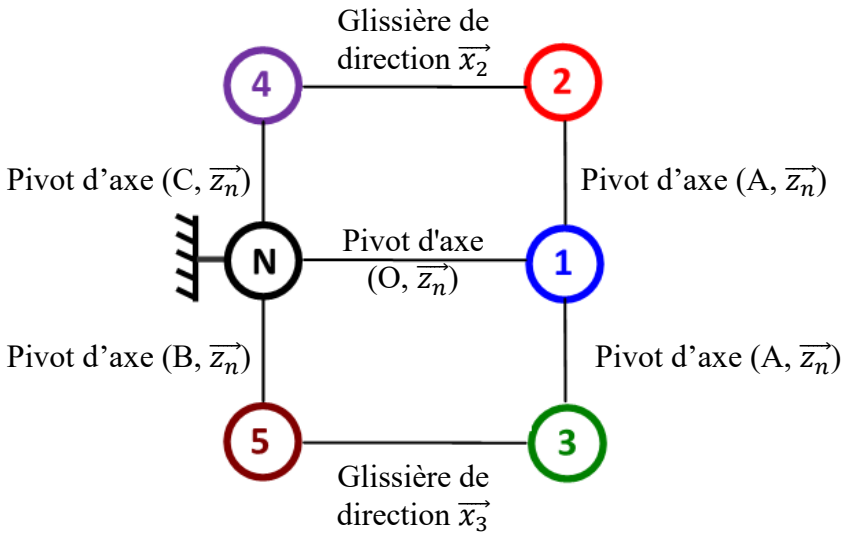
Question 10 : $\{V_{3/0}\} = \begin{Bmatrix} (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\,\overrightarrow{z_1} \\ l_1\,\dot{\theta}_1\,\overrightarrow{y_1} + l_2(\dot{\theta}_2 + \dot{\theta}_3)\,\overrightarrow{y_2} + l_3(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\,\overrightarrow{y_3} \end{Bmatrix}_M$

Question 11: $\vec{V}_{M\in 3/0}.\overrightarrow{y_0} = V_{max}$

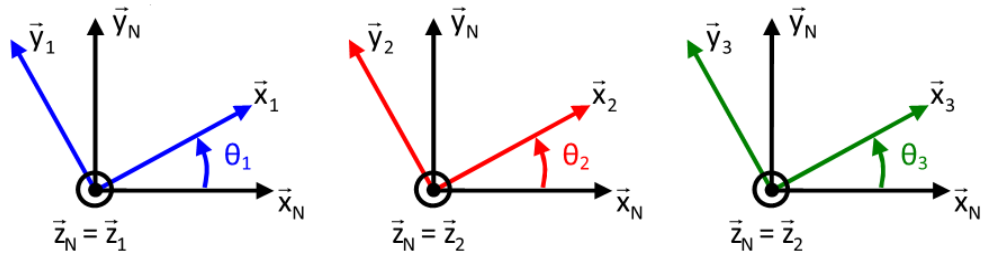
$$l_1\,\dot{\theta}_1\cos(\theta_1) + l_2(\dot{\theta}_2 + \dot{\theta}_3)\cos(\theta_1 + \theta_2) + l_3(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)\cos(\theta_1 + \theta_2 + \theta_3) = V_{max}$$

Exercice 3 : Quille d'un voilier de course Imoca 60

Question 1



Question 2



Question 3 : $\overrightarrow{0A} = R\,\overrightarrow{y_1}$

$\overrightarrow{0A} = -a\,\overrightarrow{x_n} + b\,\overrightarrow{y_n} + x_{24}(t)\,\overrightarrow{x_2}$

$\overrightarrow{0A} = a\,\overrightarrow{x_n} + b\,\overrightarrow{y_n} - x_{35}(t)\,\overrightarrow{x_3}$

Question 4 : On projette les différentes expressions de $\overrightarrow{0A}$ suivant $\overrightarrow{x_n}$ et $\overrightarrow{y_n}$:

Relation entre $x_{24}(t)$ et θ_1 :

Suivant $\overrightarrow{x_n}$:$-a + x_{24}(t)\cos(\theta_2) = -Rsin(\theta_1) <=> (x_{24}(t)\cos(\theta_2))^2 = \left(a - Rsin(\theta_1)\right)^2$

Suivant $\overrightarrow{y_n}$: $b + x_{24}(t)\sin(\theta_2) = R\,cos(\theta_1) <=> (x_{24}(t)\sin(\theta_2))^2 = (R\,cos(\theta_1) - b)^2$

En effectuant la somme de ces deux équations, on obtient :

$$x_{24}(t) = \sqrt{\left(a - R\,sin(\theta_1)\right)^2 + (R\,cos(\theta_1) - b)^2}$$
$$= \sqrt{a^2 + b^2 + R^2 - 2a.R\,sin(\theta_1) - 2b.R\,cos(\theta_1)}$$

Relation entre $x_{35}(t)$ et θ_1 :

Suivant $\overrightarrow{x_n}$:$a - x_{35}(t)\cos(\theta_3) = -R\,sin(\theta_1) <=> (x_{35}(t)\cos(\theta_3))^2 = \left(a + R\,sin(\theta_1)\right)^2$

Suivant $\overrightarrow{y_n}$: $b + x_{35}(t)\sin(\theta_3) = R\,cos(\theta_1) <=> (x_{35}(t)\sin(\theta_3))^2 = (R\,cos(\theta_1) - b)^2$

En effectuant la somme de ces deux équations, on obtient :

$$x_{35}(t) = \sqrt{\left(a + R\,sin(\theta_1)\right)^2 + (R\,cos(\theta_1) - b)^2}$$
$$= \sqrt{a^2 + b^2 + R^2 + 2a.R\,sin(\theta_1) - 2b.R\,cos(\theta_1)}$$

Question 5 :

$$x_{24}(t) = \sqrt{1{,}5^2 + 1^2 + 1^2 - 3\,sin(45°) - 2\,cos(45°)} = 0{,}845\,m$$

$$x_{35}(t) = \sqrt{1{,}5^2 + 1^2 + 1^2 + 3\,sin(45°) - 2\,cos(45°)} = 2{,}22\,m$$

Question 6 : Oui car $x_{35\,max} = 2{,}22\,m < 2{,}5\,m$ et $x_{24\,max} = 0{,}845\,m < 2{,}5\,m$

Question 7 : $\overrightarrow{OD}.\overrightarrow{y_n} = L\,\overrightarrow{y_1}.\overrightarrow{y_n} = -L\,cos\,(\theta_1) = -4{,}5\,\cos(45°) = -3{,}53\,m$

Le point D se situe à 3,53 m de profondeur

Question 8 : $\vec{\Omega}_{4/0} = \dot{\theta}_2\,\overrightarrow{z_n}$ et $\vec{\Omega}_{5/0} = \dot{\theta}_3\,\overrightarrow{z_n}$

Question 9 : $\vec{V}_{A\in 2/4} = \dot{x}_{24}(t)\,\overrightarrow{x_2}$

$\vec{V}_{A\in 4/0} = \vec{V}_{C\in 4/0} + \overrightarrow{AC} \wedge \vec{\Omega}_{4/0} = \vec{0} - x_{24}(t)\,\overrightarrow{x_2} \wedge \dot{\theta}_2\,\overrightarrow{z_n} = x_{24}(t).\dot{\theta}_2\,\overrightarrow{y_2}$

$$\vec{V}_{A\in 2/0} = \vec{V}_{A\in 2/4} + \vec{V}_{A\in 4/0} = \dot{x}_{24}(t)\,\overrightarrow{x_2} + x_{24}(t).\dot{\theta}_2\,\overrightarrow{y_2}$$

Question 10 : $\vec{V}_{A\in 3/5} = -\dot{x}_{35}(t)\,\overrightarrow{x_3}$

$\vec{V}_{A\in 5/0} = \vec{V}_{B\in 5/0} + \overrightarrow{AB} \wedge \vec{\Omega}_{5/0} = \vec{0} + x_{35}(t)\,\overrightarrow{x_3} \wedge \dot{\theta}_3\,\overrightarrow{z_n} = -x_{35}(t).\dot{\theta}_3\,\overrightarrow{y_3}$

$$\vec{V}_{A\in 3/0} = \vec{V}_{A\in 3/5} + \vec{V}_{A\in 5/0} = -\dot{x}_{35}(t)\,\overrightarrow{x_3} - x_{35}(t).\dot{\theta}_3\,\overrightarrow{y_3}$$

Question 11 : $\{V_{3/0}\} = \begin{cases} \dot{\theta}_3\,\overrightarrow{z_n} \\ -\dot{x}_{35}(t)\,\overrightarrow{x_3} - x_{35}(t).\dot{\theta}_3\,\overrightarrow{y_3} \end{cases}_A$

$\{V_{2/0}\} = \begin{cases} \dot{\theta}_2\,\overrightarrow{z_n} \\ \dot{x}_{24}(t)\,\overrightarrow{x_2} + x_{24}(t).\dot{\theta}_2\,\overrightarrow{y_2} \end{cases}_M$

Question 12 : $\vec{V}_{A\in 2/0} = \vec{V}_{A\in 1/0} = \vec{V}_{A\in 3/0}$

Question 13 : $\dot{x}_{24}(t)\,\overrightarrow{x_2} + x_{24}(t).\dot{\theta}_2\,\overrightarrow{y_2} = -\dot{x}_{35}(t)\,\overrightarrow{x_3} - x_{35}(t).\dot{\theta}_3\,\overrightarrow{y_3}$