Exercice 4

$$I_2 = \int_{-1}^{1} \frac{1}{x^2 - x + 2}\,dx = \int_{-1}^{1} \frac{1}{\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 2}\,dx$$

- On pose $t = x - \frac{1}{2}$, $\varphi(t) = t + \frac{1}{2}$, $\varphi^{-1}(t) = t - \frac{1}{2}$, $\varphi'(t) = 1$

On a donc d'après la formule de changement de variables

$$I_2 = \int_{-3/2}^{1/2} \frac{1}{t^2 + \frac{7}{4}}\,dt = \int_{-3/2}^{1/2} \frac{\frac{4}{7}}{\frac{4}{7}t^2 + 1}\,dt$$

- On pose $p = \frac{2}{\sqrt{7}}t$, $\psi(p) = \frac{\sqrt{7}}{2}p$, $\psi^{-1}(p) = \frac{2}{\sqrt{7}}p$, $\psi'(p) = \frac{\sqrt{7}}{2}$

On a donc d'après la formule de changement de variables

$$I_2 = \frac{4}{7}\int_{-3/\sqrt{7}}^{\frac{1}{\sqrt{7}}} \frac{1}{p^2 + 1} \times \frac{\sqrt{7}}{2}\,dp$$

$$= \frac{2}{\sqrt{7}}\int_{-3/\sqrt{7}}^{1/\sqrt{7}} \frac{1}{p^2 + 1}\,dp$$

$$= \frac{2}{\sqrt{7}}\left[\arctan p\right]_{-3/\sqrt{7}}^{1/\sqrt{7}}$$

$$I_2 = \frac{2}{\sqrt{7}}\left(\arctan\left(\frac{1}{\sqrt{7}}\right) - \arctan\left(-\frac{3}{\sqrt{7}}\right)\right)$$