$$I_4 = \int_0^1 x \,\text{Arctan}\, x \, dx$$

Indique que tu fais une IPP,

On pose $\begin{cases} u = \text{Arctan}\, x \\ v' = x \end{cases}$

~~on en déduit~~:

$$\int_0^1 u v' = [uv]_0^1 - \int_0^1 u' v$$

et $\begin{cases} u' = \dfrac{1}{1+x^2} \\ v = \dfrac{1}{2}x^2 \end{cases}$

pas top car tu as un choix pour v.

$$I_4 = \left[\text{Arctan}\, x \cdot \frac{1}{2}x^2\right]_0^1 - \int_0^1 \frac{1}{2}x^2 \times \frac{1}{1+x^2}\, dx$$

$$I_4 = \left[\text{Arctan} \cdot \frac{1}{2}x^2\right]_0^1 - \frac{1}{2}\int_0^1 \left(\frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\right) dx$$

$$I_4 = \left[\text{Arctan}\, x \cdot \frac{1}{2}x^2\right]_0^1 - \frac{1}{2}[x]_0^1 + \frac{1}{2}[\text{Arctan}\, x]_0^1$$

$$I_4 = 2\frac{\pi}{8} - \frac{1}{2}$$

$$= \boxed{\frac{\pi}{4} - \frac{1}{2}}$$

TB

$$I_4 = \int_1^{64} \frac{1}{\sqrt{x}+\sqrt[3]{x}} = \int_1^{64} \frac{1}{x^{1/2}+x^{1/3}}\,dx$$

CDV

On pose $x = t^6$ d'où $\varphi(t) = t^6$ $\varphi'(t) = 6t^5$

$\varphi^{-1}(t) = \sqrt[6]{t} = t^{1/6}$

$$I_4 = \int_1^{2} \frac{1}{t^3+t^2} \times 6t^5\,dt$$

Factorisant par t^2

$$I_4 = 6\int_1^{2} \frac{t^5}{t^3+t^2}\,dt = 6\int_1^{2} \frac{t^3}{t+1}\,dt$$

Division euclienne de t^3 par $t+1$:

$$t^3 = (t+1)(t^2-t+1) - 1$$

$$I_4 = 6\int_1^{2} \frac{(t^2-t+1)(t+1)-1}{t+1}\,dt$$

$$I_4 = 6\int_1^{2} t^2-t+1\,dt - 6\int_1^{2} \frac{1}{t+1}\,dt$$

$$= \underbrace{6\left[\frac{1}{3}t^3-\frac{1}{2}t^2+t\right]_1^2}_{11} - 6\left[\ln(t+1)\right]_1^2$$

$$= 11 + 6\ln(2) - 6\ln(3) \cancel{\Rightarrow} \boxed{I_4 = 11 - \cancel{\ln}(2/3)}$$

$+6\ln\frac{2}{3}$