Fiche n° 2. Signaux

Réponses

2.1 a) .... $-\sin(\alpha)$

2.1 b) .... $-\sin(\alpha)$

2.1 c) .... $\cos(\alpha)$

2.1 d) .... $\cos(\alpha)$

2.2 a) .... $2\cos(2t)$

2.2 b) ... $-2\sin(t+4)\cos(t+4) = -\sin(2t+8)$

2.2 c) .... $\cos^2(t) - \sin^2(t) = \cos(2t)$

2.3 a) .... $2A\cos\left(\dfrac{\omega_1-\omega_2}{2}t\right)\cos\left(\dfrac{\omega_1+\omega_2}{2}t\right)$

2.3 b) .... $2A\sin\left(\dfrac{\omega_2-\omega_1}{2}t\right)\sin\left(\dfrac{\omega_1+\omega_2}{2}t\right)$

2.4 .... $A\sin(\varphi)\cos(\omega t) + A\cos(\varphi)\sin(\omega t)$

2.5 a) .... Courbe 2

2.5 b) .... Courbe 4

2.5 c) .... Courbe 3

2.5 d) .... Courbe 1

2.6 .... (c)

2.7 a) .... 1,5 V

2.7 b) .... $\dfrac{\pi}{2}$ rad

2.7 c) .... 2 s

2.7 d) .... 0,5 Hz

2.7 e) .... π rad $\cdot$ s^{-1}

2.8 a) .... En retard

2.8 b) .... $\varphi < 0$

2.8 c) .... $-\dfrac{2\pi}{3}$ rad

2.9 a) .... $u_3(t)$

2.9 b) .... $u_1(t)$

2.9 c) .... $u_2(t)$

2.10 a) 0

2.10 b) .... $\dfrac{U_0}{\sqrt{2}}$

2.11 a) .... 1,5 V

2.11 b) .... $\sqrt{3}$ V

2.12 a) .... $\dfrac{U_0}{2}$

2.12 b) .... $\dfrac{U_0}{\sqrt{2}}$

2.13 a) .... 1,7 km

2.13 b) .... 5,7 µs

2.13 c) .... oui

2.14 .... 18 km/h

2.15 a) .... 1,6 s

2.15 b) .... 48 cm

2.15 c) .... $2\sin(3{,}9t - 13x + 0{,}3\pi)$

Corrigés

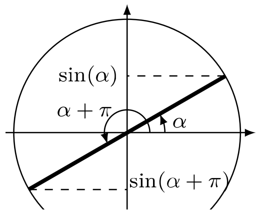
2.1 a)

En utilisant le cercle trigonométrique, on voit directement que

$$\sin(\alpha+\pi) = -\sin(\alpha).$$

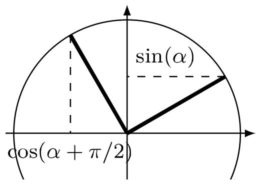
Remarquons qu'on peut également utiliser les formules trigonométriques :

$$\sin(\alpha+\pi) = \sin(\alpha)\cos(\pi) + \sin(\pi)\cos(\alpha) = -\sin(\alpha).$$



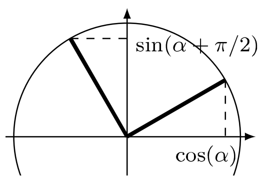
2.1 b)

On a $\cos\left(\alpha + \dfrac{\pi}{2}\right) = -\sin(\alpha)$.



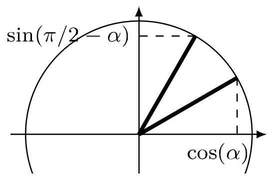
2.1 c)

On a $\sin\left(\alpha + \dfrac{\pi}{2}\right) = \cos(\alpha)$.



2.1 d)

On a $\sin\left(\dfrac{\pi}{2} - \alpha\right) = \cos(\alpha)$.



2.3 a) On somme les formules trigonométriques :

$$\begin{cases} \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \\ \cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b) \end{cases} \quad \text{pour obtenir} \quad \cos(a+b) + \cos(a-b) = 2\cos(a)\cos(b).$$

On a

$$\begin{cases} a+b = \omega_1 t \\ a-b = \omega_2 t \end{cases} \iff \begin{cases} a = \dfrac{\omega_1+\omega_2}{2} t \\ b = \dfrac{\omega_1-\omega_2}{2} t. \end{cases}$$

On en déduit

$$A\cos(\omega_1 t) + A\cos(\omega_2 t) = 2A\cos\left(\frac{\omega_1+\omega_2}{2}\, t\right)\cos\left(\frac{\omega_1-\omega_2}{2}\, t\right).$$

Ainsi, $C = 2A$, $\Omega = \dfrac{\omega_1+\omega_2}{2}$ et $\omega = \dfrac{\omega_1-\omega_2}{2}$ conviennent.
